

Interpretable KELM data-driven model for the prediction and monitoring of arch dam behavior

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ABSTRACT: This paper presents a kernel extreme learning machine (KELM)-based nonlinear data-driven model for the dam behavior (i.e., radial displacement and seepage) prediction, where the model hyperparameters are determined using particle swarm algorithm (PSO) and internal cross-validation to overcome overfitting. The model inputs are composed of the reservoir water height, measured temperature, rainfall, and time-related variables. The global sensitivity analysis coupled with the KELM model is proposed for the model interpretation. The warning thresholds of the arch dam radial displacement and seepage can be determined via the residuals of training samples and the confidence interval method.

1 INTRODUCTION

Concrete dams play important roles in the social and economic fields by flood control, power generation, water supply, and irrigation. During the service period, dams are subjected to a variety of operational and environmental loads and occasionally encounter some unconventional events or extreme loads (such as excessive flooding, droughts, earthquakes, etc.). Moreover, the overall performance of the concrete structures may decrease over time due to age-related deterioration, hydraulic erosion, and other factors. If a dam is not well managed and maintained, failures may occur, leading to economic and life losses in reservoir regions.

The displacement and seepage are two critical indicators that can intuitively reflect the operational status of a dam. With the rapid development of artificial intelligence (AI) since the end of the past century, there has been growing interest in adopting machine learning (ML) methods in dam engineering. Many ML methods have been adopted for dam behavior prediction and monitoring, such as auto-associative neural networks (Kao et al., 2013), support vector regression (Rankovic et al., 2014), boosted regression trees (Salazar et al., 2015), random forest (Belmokre et al., 2019; X. Li et al., 2019), Gaussian process regression (Lin et al., 2019), and long short-term memory network (Liu et al., 2020).

This study establishes a kernel extreme learning machine (KELM)-based nonlinear data-driven model to predict the dam displacement and seepage. The model hyperparameters are optimized using a particle swarm algorithm (PSO) and cross-validation. To mine the influencing factors of model inputs and provide support for decision-making, the global sensitivity analysis coupled with the KELM model is implemented for the model interpretation. The warning thresholds of the dam radial displacement and seepage are determined using the confidence interval method.

The rest of the paper is summarized as follows: Section 2 describes the statistical model of dam behavior and inputs. The theory of the KELM prediction model, warning thresholds of dam behavior, and model interpretation method are then introduced in Section 3. Results and discussion are presented in Section 4. Finally, Section 5 summarizes the findings.

2 STATISTICAL MODEL OF DAM BEHAVIOR

2.1 Statistical model of dam displacement

Displacement (denoted by δ) is a key indicator for evaluation of the dam behavior. In general, the displacements of the arch dam are assumed to be dependent on hydrostatic load, temperature, and time, which can be quantitatively interpreted and approximated as:

$$\delta = \delta_H + \delta_T + \delta_\theta \quad (1)$$

where δ_H , δ_T , and δ_θ represents the hydrostatic component, temperature component and time component, respectively.

Under the action of water pressure, hydrostatic component δ_H can be described by a polynomial function consisting of reservoir water height H and coefficients a_i ($i=0 \sim 4$) (Mata, 2011):

$$\delta_H = a_0 + a_1H + a_2H^2 + a_3H^3 + a_4H^4 \quad (2)$$

Temperature component δ_T describes the displacement caused by the temperature changes in bedrock and dam concrete. The temperature variation of the dam is mainly influenced by changes of air temperature. Meanwhile, there is a hysteresis effect between the air temperature and the dam internal temperature. Therefore, if the air temperatures are available and continuous, the temperature component δ_T can be quantitatively represented by a polynomial function consisting of segmented air temperature T_{A-B} (Kang et al., 2019), as:

$$\delta_T = \sum_{i=1}^n b_i T_{A-B} \quad (3)$$

where T_{A-B} denotes the average ambient temperatures A to B days before the day of observation, b_i ($i=1 \sim 6$) are coefficients. In this paper, T_0 , T_{1-2} , T_{3-7} , T_{8-15} , T_{16-30} , and T_{31-60} are selected as the temperature factors. It is noted that T_0 represents the temperature of the observation day.

Time component δ_θ reflects the irreversible deformation of the dam body or dam foundation toward a certain direction over time. According to the previous research (Lin et al., 2019; Y. Q. Shi et al., 2018), different and strictly monotone functions can be used for modeling the time component δ_θ , as:

$$\delta_\theta = c_1\theta + c_2\ln\theta + c_3(1 - e^{-\theta}) + c_4(\theta/\theta + 1) \quad (4)$$

where $\theta = t/100$, and t denotes number of days since the beginning of the analysis, c_1 , c_2 , and c_3 are coefficients.

Thus, the statistical model of arch dam displacement contains the 14 input factors, and the expression is as follows:

$$\begin{aligned} \delta = & a_0 + a_1H + a_2H^2 + a_3H^3 + a_4H^4 + T_0 + T_{1-2} + T_{3-7} + T_{8-15} + T_{16-30} + T_{31-60} + \\ & c_1\theta + c_2\ln\theta + c_3(1 - e^{-\theta}) + c_4(\theta/\theta + 1) \end{aligned} \quad (5)$$

2.2 Statistical model of dam seepage

Excluding the hydrostatic load, temperature and time effects, the seepage (denoted by s) of the arch dam is also dependent on the rainfall effect, and the seepage response can be quantitatively interpreted and approximated by the following equation:

$$S = S_H + S_T + S_R + S_\theta \quad (6)$$

where S_H , S_T , and S_θ represent the hydrostatic component, temperature component, and time component, respectively, sharing the same form as shown in Equation (2)~(4). S_R denotes the rainfall component. Considering the lag effect between the rainfall and the external seepage changes, a polynomial function consisting of segmented rainfall factors R_{A-B} are utilized to simulate the rainfall component:

$$S_R = \sum_{i=1}^m d_i R_{A-B} \quad (7)$$

where R_{A-B} denotes the average rainfall A to B days before the response day of the observation. In this paper, R_0 , R_{1-2} , R_{3-7} , R_{8-15} , R_{16-30} , and R_{31-60} are selected as the segmented rainfall factors, where R_0 denotes the rainfall of the observation day.

Thus, the statistical model of arch dam seepage contains the 20 input factors, and the expression is as follows:

$$\begin{aligned} S = & a_0 + a_1H + a_2H^2 + a_3H^3 + a_4H^4 + T_0 + T_{1-2} + T_{3-7} + T_{8-15} + T_{16-30} + T_{31-60} + \\ & R_0 + R_{1-2} + R_{3-7} + R_{8-15} + R_{16-30} + R_{31-60} + c_1\theta + c_2\ln\theta + c_3(1 - e^{-\theta}) + c_4(\theta/\theta + 1) \end{aligned} \quad (8)$$

3 METHODOLOGY OF DAM BEHAVIOR PREDICTION AND WARNING

3.1 Optimized kernel extreme learning machine

Extreme learning machine (ELM) is an extension algorithm of the single layer feedforward network (SLFN) that can be used for regression, classification, and clustering (Huang et al., 2006). As opposed to the traditional artificial neural network based on gradient descent learning algorithm, ELM has a stochastic nature. It randomly assigns the input weights and the hidden layer biases, and then keep them fixed without iteratively tuning. In recent years, a novel variant of ELM called kernel extreme learning machine (KELM) has been proposed by (Huang et al., 2012), which integrates the advantages of ELM and kernel trick. The KELM was shown to achieve a better prediction performance and stability than prototype ELM with less computational cost (Ding et al., 2013).

The output of the ELM for generalized SLFNs can be written as

$$F_j = \sum_{i=1}^N \beta_i \mathbf{h}(a_i \cdot x_j + b_i), \quad j = 1, \dots, N \quad (9)$$

where a_i denotes the weight vector linking i th hidden node and the input nodes; β_j presents the weight vector connecting j th hidden node and the output nodes; b_i is the threshold of i th hidden node. \mathbf{h} refers to the activation functions.

The training goal is to find the best output weight β , which can be computed by the least square method:

$$\beta = \mathbf{H}^\dagger \mathbf{T} \quad (10)$$

where \mathbf{H}^\dagger denotes the Moore-Penrose (MP) generalized inverse of the hidden layer output, and $\mathbf{T} = [t_1, t_2, \dots, t_N]^T$ presents the target vector.

For complex prediction task, hidden layer feature mapping is typically unknown. Thus, the kernel function is introduced to replace the feature mapping function. On the basis of the orthogonal projection method, the MP generalized inverse matrix \mathbf{H}^\dagger can be calculated by $\mathbf{H}^\dagger = \mathbf{H}^T (\mathbf{H}\mathbf{H}^T)^{-1}$, and the output weight β can be computed by adding a positive constant, $1/C$. Therefore, the output function of KELM can be briefly described given by

$$F(x) = \mathbf{h}\beta = \mathbf{h}(x)\mathbf{H}^\dagger \left(\frac{\mathbf{I}}{C} + \mathbf{H}\mathbf{H}^T \right)^{-1} \mathbf{T} = \begin{bmatrix} K(x_1, x) \\ \vdots \\ K(x_N, x) \end{bmatrix}^T \left(\frac{\mathbf{I}}{C} + \Omega_{ELM} \right)^{-1} \mathbf{T} \quad (11)$$

where $K(x_i, x)$ is the kernel function and should satisfy the Mercer condition. In this study, Gaussian kernel is used in the form of $K(x_i, x) = \exp(-\|x_i - x\|^2 / 2\gamma^2)$. Therefore, the main parameters of KELM herein are regularization parameter C , and kernel parameters γ .

The performance of the KELM model is controlled by hyperparameters C and γ . To make sure the model brings good generalization and robust performance, particle swarm optimizer (PSO) (Y. H. Shi et al., 1998) was combined with 3-fold cross-validation to determine the optimal parameters. In 3-fold cross-validation, the training data is divided into an internal validation set and an internal training set. For the PSO algorithm, the population size is set to 20, and the maximal iteration is set to 20 as the stopping criteria. In each iteration, P is the dimensions of the hyperparameters to be optimized, the position vector $\mathbf{X}_i = [x_i^1, x_i^2, \dots, x_i^P]$ and the velocity vector $\mathbf{V}_i = [v_i^1, v_i^2, \dots, v_i^P]$ are updated once by the following equations:

$$\begin{aligned} v_i^p &= v_i^p + c_1 \cdot rand_1^p \cdot (pbest_i^p - x_i^p) + c_2 \cdot rand_2^p \cdot (nbest_i^p - x_i^p) \\ x_i^p &= x_i^p + v_i^p \end{aligned} \quad (12)$$

where c_1 and c_2 are two acceleration coefficients with the values are set as 2.0. $rand_1^p$ and $rand_2^p$ denote the two random numbers generated independently within $[0, 1]$. $pbest_i^p$ denotes the position with the best-known fitness of the i th particle, and $nbest_i^p$ represents the best global position in $pbest_i^p$.

In each iteration, the error function of PSO is evaluated by mean squared error of prediction ($MSEP_{cv,k}$), as shown in Equation (13):

$$MSEP_{cv,k} = \frac{1}{n} \sum_{k=1}^K \sum_{i=1}^n (\hat{y}_i - y_i)^2 \quad (13)$$

where n represents the number of samples, K represents the number of folds. \hat{y}_i is the predicted value of the internal validation samples, y_i is the measured value of the internal validation samples.

3.2 Warning thresholds of dam behavior

The confidence interval method (B. Li et al., 2019) is used for dam behavior monitoring and determine the warning thresholds. If the measured value falls within the interval range, it is regarded as a safe value. Otherwise, it is regarded as anomalous and may raise alarm. The expression of the confidence interval (CI) is given in Equation (14):

$$CI = [(\hat{y} - \alpha\sigma), (\hat{y} + \alpha\sigma)]; \quad \sigma = \sqrt{\frac{\sum_{i=1}^{N_i} (e_i - \bar{e})^2}{N_i - 1}} \quad (14)$$

where \hat{y} represents the fitted or predicted value of dam behavior, σ is the standard deviation, N_t represents the number of the training samples. $e_i = (y_i - \hat{y}_i)$, where \bar{e} is the mean value of e_i , y_i is the measured value of dam behavior, and \hat{y}_i is the fitted value of dam behavior.

It is noted that abnormal values may not be identified if the confidence interval is relatively wide. In contrast, if the range is too narrow, many values may be deemed as abnormal mistakenly (Wu, 2003), thereby resulting in many false positives. In general, the effectiveness of confidence interval is influenced by the input selection, performance of the KELM model, and hyperparameters tuning, which need to be determined with caution. Considering the dam status and risk level, the extreme case scenario is considered, the significance level is set as 1%, and therefore, $\alpha \approx 2.58$.

3.3 Global sensitivity analysis for model interpretation

KELM predictive model contains the disadvantages of black box characteristics, and the trained data-driven model is typically difficult to be understood. Inspired by (Chen et al., 2020; Cortez et al., 2013) pioneer work, we combined global sensitivity analysis (GSA) with the PSO-KELM to interpretate the dam behavior prediction model. This method allows us to compute the relative importance of input variables or any group combination of them. The main idea of the GSA is to hold all input variables at given value except the specific variable to be computed, and then calculate the output weight of the corresponding input by the available formula. The detailed procedure of the GSA is described in Table 1, where m and n are the number of input attributes and samples, respectively. \mathcal{M} is the number of subgroups (each subgroup contains at least one input attribute). $\mathbf{X}^{(k)}$ is the generated meta-inputs by holding all input variables at their mean values except k th attributes, and $k \leq \mathcal{M}$. $\hat{\mathbf{y}}^{(k)}$ represents the obtained output via inputting the $\mathbf{X}^{(k)}$ to the trained model, and \bar{y} denotes the median value of the measured leakage $\mathbf{Y}^{(n \times 1)}$. In principle, the proposed GSA can be applied to any supervised machine learning algorithm for regression tasks.

Table 1. Global sensitivity analysis for model interpretation.

Inputs: Training samples $\mathbf{X}^{(n \times m)}, \mathbf{Y}^{(n \times 1)}$

Outputs: Relative importance **RI**

Divide \mathbf{X} into k groups of attributes, $k \in [1, \mathcal{M}]$

for $k = 1, \dots, \mathcal{M}$

Build $\mathbf{X}^{(k)}$ by holding each input variables at their mean values except k th attributes

$\hat{\mathbf{y}}^{(k)} = f(\mathbf{X}^{(k)})$

$z_k = \sum_{i=1}^n |\hat{y}_i^{(k)} - \bar{y}| / (n-1)$

end for

for $k = 1, \dots, \mathcal{M}$

$RI_k = z_k / \sum_{k=1}^{\mathcal{M}} z_k \times 100\%$

end for

4 RESULTS AND DISCUSSION

4.1 Case study

4.1.1 Brief introduction of dam project

The case study of the benchmark is a double curvature arch dam called Dam_EDF, which is located in the south of France. The dam was constructed between 1957 and 1960. The maximum dam height above the foundation is about 45m, with the crest length being 166 m. To monitor the dam service status, the dam is equipped with a comprehensive monitoring system and instruments. Figure 1 presents the illustrations of the dam project.

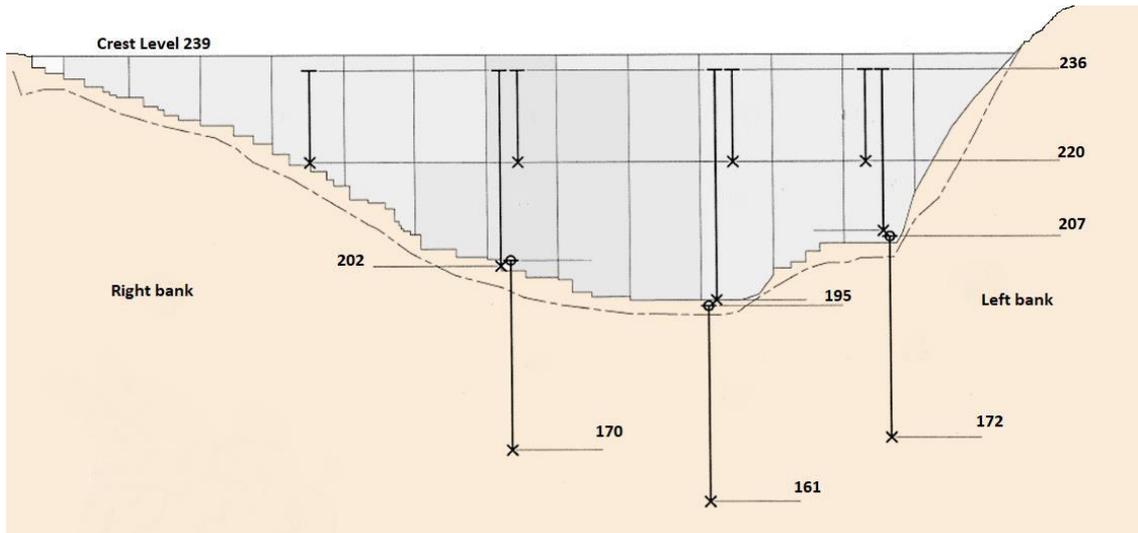


Figure 1. Downstream view of Dam_EDF.

4.1.2 Data collection

In this benchmark, the radial dam displacement and seepage are used for analysis. For the radial dam displacement (unit is mm), the measurements of pendulums on the Central Block (CB) are provided for analysis, where CB2 is the radial displacement between the altitudes 236 m (dam crest) and 196 m (dam toe), while CB3 is the radial displacement in the foundation between the altitudes 195 m and 161 m. For the seepage (unit is $L \cdot \text{min}^{-1}$), the flowrate is measured using a weir located in the gallery at the downstream dam toe. The time series of dam behavior data are provided from 2000 to 2012.

The corresponding ambient data includes the water level, temperature, and rainfall (see Figure 2 ~ Figure 4). The water level of the reservoir is collected per day. Since Dam_EDF is located on the top of a glacial threshold, the reservoir water height is 0 once the water level is lower than +196 m. The air temperature is not measured at the dam location, therefore, the provided calculated temperature called 'T_b' is used herein for temperature factors generation. 'T_b' is calculated by interpolation from several air temperature measuring stations. The interpolation takes into account the altitude of the dam and is calculated on a mesh of 1 square kilometer. Daily rainfall precipitation is collected from a rain gauge located about 5 km from Dam_EDF. The time series of the ambient data is provided from 1995 to 2017. It is noted that the provided data of the benchmark is automatically checked, and there is no need for any further cleaning.

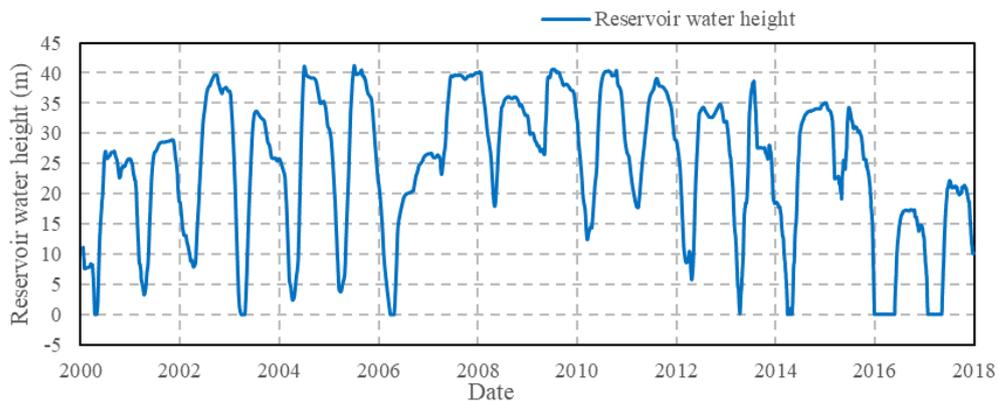


Figure 2. Time series of the reservoir water height.

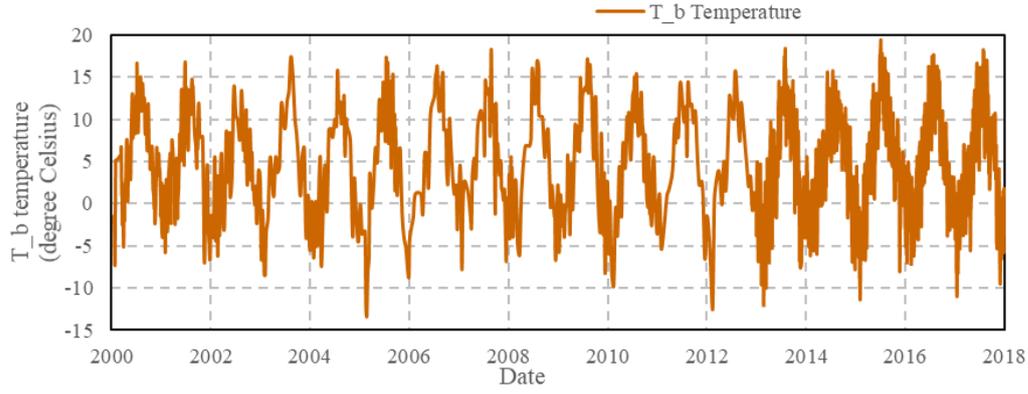


Figure 3. Time series of the T_b air temperature.

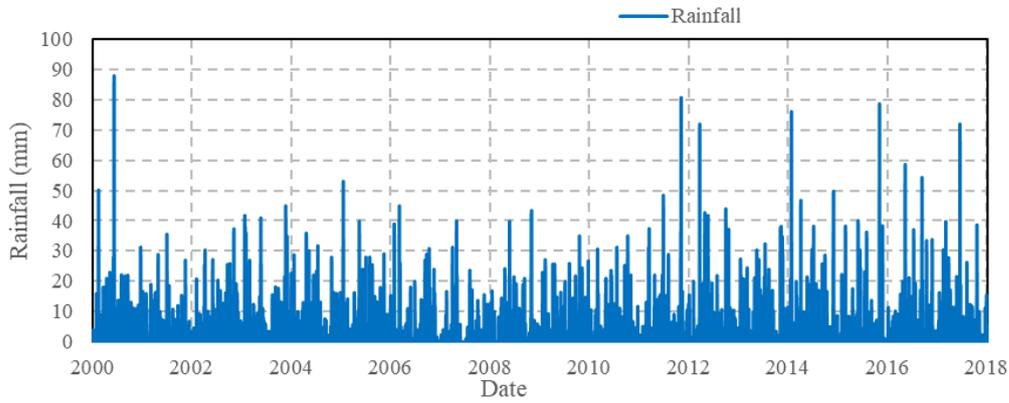


Figure 4. Time series of the daily rainfall.

4.2 Model calibration and prediction

Based on the introduces in Section 2, the model input variables of dam displacement \mathbf{x}_δ and seepage \mathbf{x}_s are shown as follows:

$$\mathbf{x}_\delta = \left\{ H, H^2, H^3, H^4, T_0, T_{1-2}, T_{3-7}, T_{8-15}, T_{16-30}, T_{31-60}, \theta, \ln \theta, (1 - e^{-\theta}), (\theta/\theta + 1) \right\} \quad (15)$$

$$\mathbf{x}_s = \left\{ H, H^2, H^3, H^4, T_0, T_{1-2}, T_{3-7}, T_{8-15}, T_{16-30}, T_{31-60}, R_0, R_{1-2}, R_{3-7}, R_{8-15}, R_{16-30}, R_{31-60}, \theta, \ln \theta, (1 - e^{-\theta}), (\theta/\theta + 1) \right\} \quad (16)$$

where the time component factors are generated.

Prior to model implementation, all the inputs should be normalized within the range of [0,1] by Equation (17), where x_i is i th individual variable in input matrix \mathbf{x} .

$$m(x_i) = \frac{x_i - \min(\mathbf{x})}{\max(\mathbf{x}) - \min(\mathbf{x})} \quad (17)$$

The measured radial displacement and seepage from 2000-01-19 to 2012-12-31 are utilized for model training and calibration, and the rest measured radial displacement from 2013-01-01 to 2017-12-31 are utilized for validation of prediction performance. By implementing the method introduced in Section 3.1, the hyperparameters C and γ are tuned within the range of (0,1000] and (0,10], respectively. The obtained hyperparameters for KELM prediction model are listed in Table 2. The calibration and prediction results of CB2 displacement, CB3 displacement, and seepage are shown in Figure 5 ~ Figure 7, respectively.

Table 2. The hyperparameters of PSO-KELM data-driven models.

Hyperparameters	CB2 displacement	CB3 displacement	Seepage
C	748.529	165.711	34.467
γ	3.453	0.650	0.591

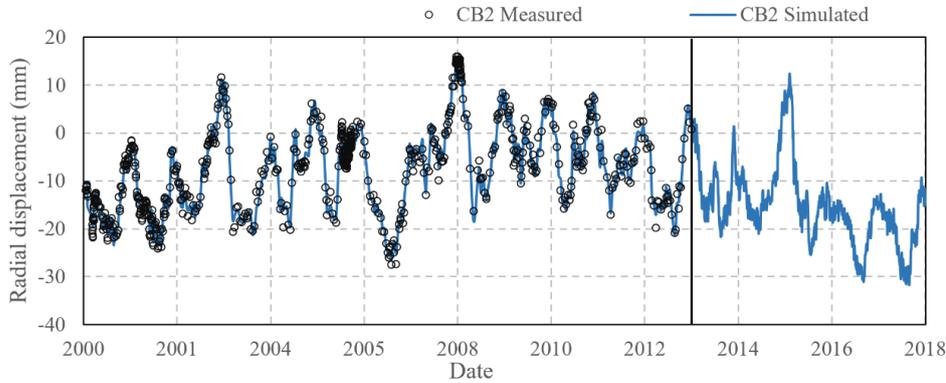


Figure 5. Performance of the PSO-KELM model for CB2 displacement simulation. (The number of training sample is 688)

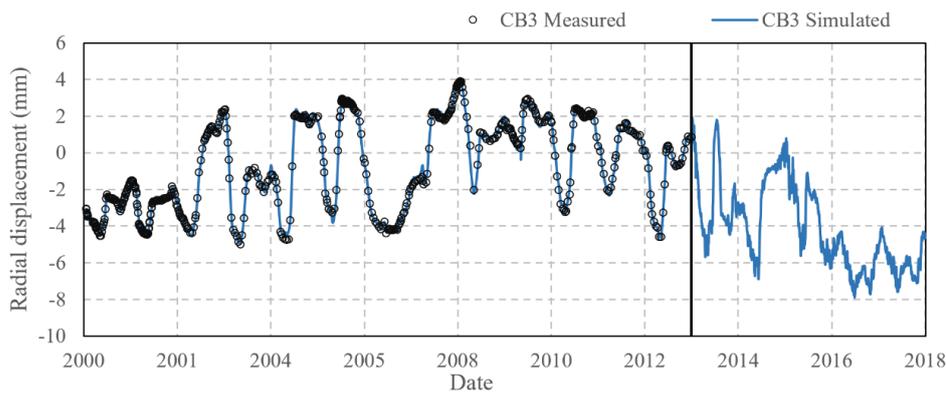


Figure 6. Performance of the PSO-KELM model for CB3 displacement simulation. (The number of training sample is 681)

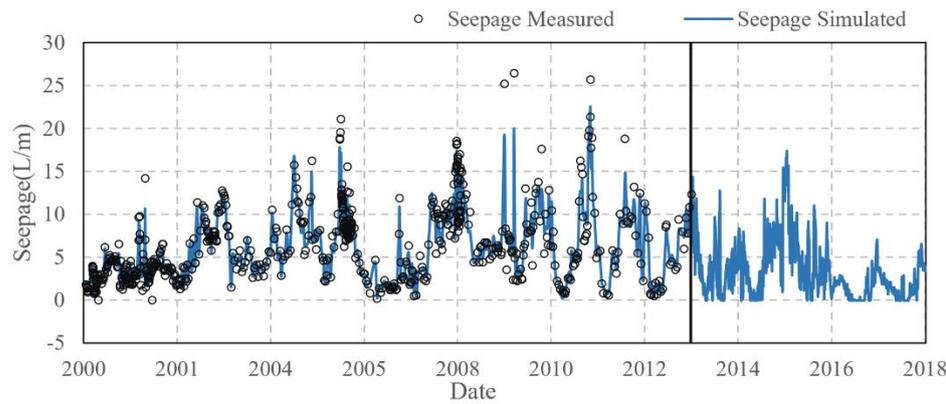


Figure 7. Performance of the PSO-KELM model for seepage simulation. (The number of training sample is 661)

The calibration results of displacement and seepage are validated in terms of the mean absolute error and normalized root mean squared error (NRMSE), see Table 3.

$$MAE = \frac{1}{N} \sum_{i=1}^N |Y_i - P_i| \quad (18)$$

$$NRMSE = \frac{\sqrt{\sum_{i=1}^N (Y_i - P_i)^2 / N}}{Y_{\max} - Y_{\min}} \quad (19)$$

where N is the number of time stamps in the corresponding period, Y_{\max} and Y_{\min} are maximum and minimum measured value of dam behavior in the corresponding period, Y_i denotes the measured value of dam behavior, and P_i denotes the simulated value of dam behavior.

From the obtained results shown above, the PSO-KELM model provides satisfactory fitting performance of dam behavior. Most of the measured value are within the interval range except the very few measurements at the peak value.

Table 3. The metrics of calibration results.

Metrics	CB2 displacement	CB3 displacement	Seepage
MAE	0.771	0.087	0.576
NRMSE	0.023	0.015	0.035

4.3 Model interpretation

The relative importance of input factors via PSO-KELM model coupled with the GSA method is depicted in Figure 8, where the relative importance of dominated input variables in each component is shown by the bar plot. It is found that the hydrostatic component plays a crucial role in the state of dam displacement and seepage, followed by the time component and temperature component. However, the seepage of the dam is not sensitive to the rainfall variables, indicating that the rainfall component of the dam seepage is negligible. It must be explained with great caution that the results via the GSA herein are not able to assess what absolute extent each input accounts for dam behavior, but only the relative sensitivity degree of the dam behavior to each input variable.

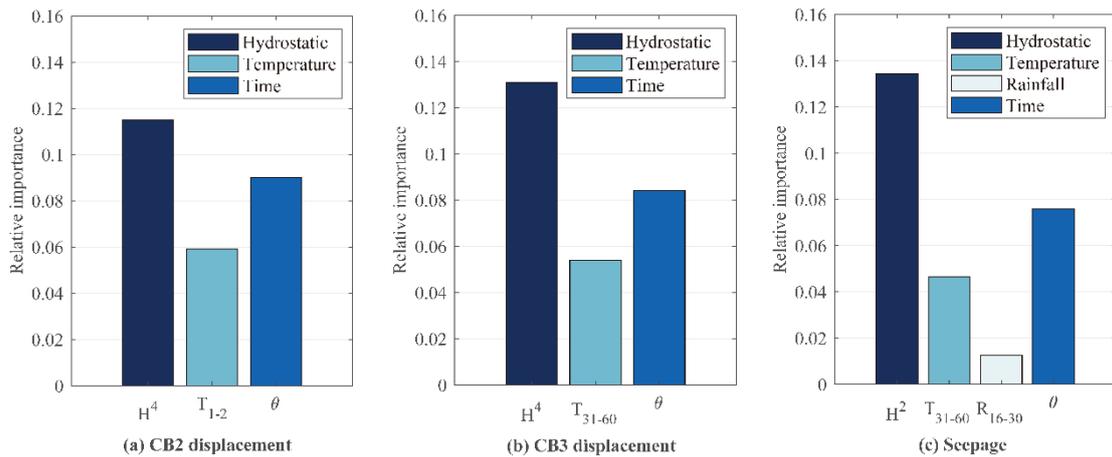


Figure 8. Relative importance of input factors via PSO-KELM model coupled with GSA method: (a) CB2 displacement, (b) CB3 displacement, (c) Seepage

5 CONCLUSIONS

In this paper, we proposed an interpretable PSO-KELM data-driven model for the prediction and monitoring of arch dam behavior (i.e., displacement and seepage). The effects of the reservoir water height, daily temperature, daily rainfall, and time were considered for inputs factors generation. By combing the PSO and 3-folds cross-validation with KELM, the hyperparameters were adaptively determined to guarantee the model generalization. Benefitting from the powerful non-linear mapping and interpretable capability, the model provides satisfactory fitting performance and reasonable interpretations. It could be learned from the results that the hydrostatic component accounts most for the dam displacement and seepage, while the time component came second. The rainfall component of seepage was negligible.

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