Behavior prediction of a concrete arch dam

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ABSTRACT: The assessment of the structural stability and the behavior of the dam during construction and service period is of vital importance. In the paper are presented acknowledgments from the numerical analysis of concrete arch dam under static action, by application of numerical methods, based on Finite Element Method, with the code SOFiSTiK. The aim of the task is to predict the dam behavior, that includes calibration (based on monitoring data) and prognosis stage (short-term and long-term) focusing on variables such as radial displacements, crack displacements, piezometric levels and seepage. Coupled thermo-mechanical analysis and seepage analysis in time domain were carried out the for calibration and prognosis stage of the specified variables. The key conclusion from the numerical experiment is that the dam behavior, with adopted geometry and material, is within the expected mode.

1 INTROODUCTION

The dams, having in consideration their importance, dimensions, complexity of the problems that should be solved during the process of designing and construction along with the environmental impact are lined up in the most complex engineering structures (Tanchev, 2014; Novak & all., 2007). The assessment of the structural stability and the behavior of the dam during construction, at full reservoir and during the service period is of paramount importance for such structures.

Static stability of concrete dams is confirmed with analysis (research) of the response of the structure (dam) under action of static loads (Mitovski & all, 2015, Mitovski & all, 2017a, Mitovski & all, 2017b]. In this paper are systemized acknowledgments from the linear and nonlinear static numerical analysis of concrete arch dam, obtained with application of numerical methods, based on Finite Element Method, with the code SOFiSTiK. Namely, here below will be illustrated output data from the numerical experiment for prediction behavior of the arch dam Dam EDF, located in France. The aim of the task is to predict the dam behavior, that includes calibration (based on monitoring data) and prognosis stage (short-term and long-term) focusing on variable such as radial displacements (two pendulums in central block of the dam), crack displacements (sensor at the rock-concrete interface), piezometric levels (vibrating wire piezometers at the rock-concrete interface) and seepage (weir at the downstream of the dam).

2 CASE STUDY

The analyzed dam is located in southern France, constructed in period 1957-1960. It is a case of double curvature arch dam, with asymmetric shape due to the valley formation (Fig. 1). The dam foundation is laminated metamorphic slate with high compressive strength, with present anisotropy in the left bank. The dam height above the foundation si H=45 m, with crest thickness of 2 m and base thickness of 6 m.

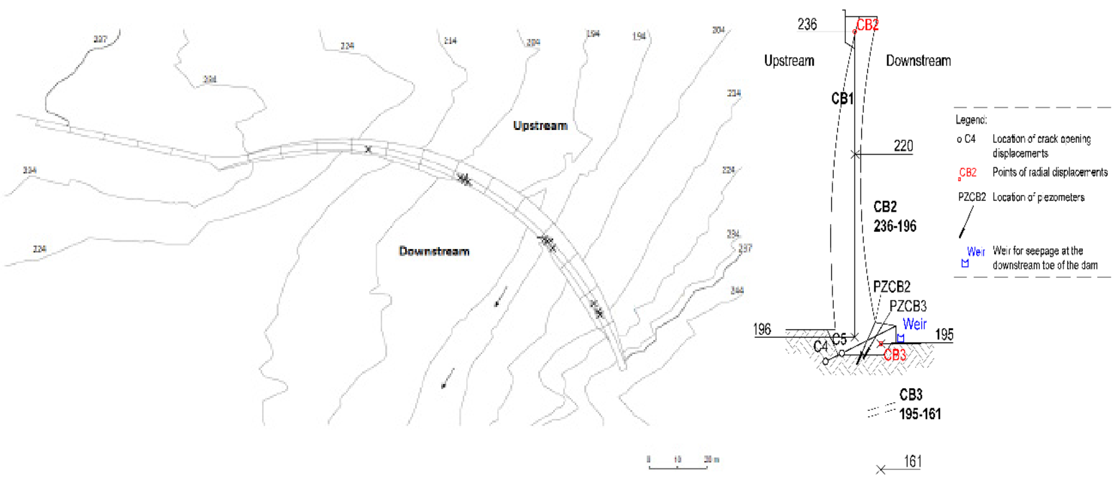

Figure 1. Layout of the dam (left) and central block section with display of monitoring instruments (right).

3 NUMERICAL MODEL OF THE DAM

The numerical analysis of Dam EDF is carried out by application of program SOFiSTiK, produced in Munich, Germany. The program is based on finite element method and has possibilities for complex modeling of the structures and simulation of their behavior. It also has possibility in the analysis to include certain specific phenomena, important for realistic simulation of dam's behavior, such as: discretization of the dam and foundation taking into account the irregular and complex geometry of the structure, simulation of stage construction, simulation of contact behavior by applying interface elements and etc. in order to assess the dam behavior and evaluate its stability. The program SOFiSTiK in its library contains and various standards and constitutive laws (linear and non-linear) for structures analysis.

The numerical experiment includes following steps, typical for this type of analysis: (1) choice of material properties and constitutive laws (concrete and rock); (2) discretization of the dam and the rock foundation and (3) simulation of the dam behavior for the typical loading states (as required in the topic formulation).

3.1 *Material properties*

The linear material properties for the dam body (concrete) and the foundation (rock) are systematized in Table 1. The specified parameters are adopted according to Theme A formulation (Malm & all, 2021) as well and previous carried out analysis and reference literature (Desai & Gallagher, 1984, SOFiSTiK, 2022, EC 2, 1992, Mitovski, 2015, USBR, 1977).

Table 1. Material parameters.

Zone		dam body (concrete)	rock	Comment
γ_{spec}	kN/m^3	24.0	27.0	Unite weight
k_s	m/s		2.0e-05	Permeability coefficient
ν		0.350	0.450	Poisson coefficient
Alpha	1/C°	7.0E-06		Thermal expansion coefficient
E	GPa	22	3	Young's modulus of elasticity

Additionally, for carrying out of non-linear analysis of the dam in order to calibrate and predict the relative distance values at interface dam-foundation is applied non-linear constitutive law for concrete based on elasto-plastic material law Lade with non-associated flow rule from SOFiSTiK library of materials (Table 2).

Table 2. Non-linear material parameters for concrete.

Zone		dam body (concrete)	Comment
γ_{spec}	kN/m^3	24.0	Unit weight
ν		0.350	Poisson coefficient
Alpha	1/C°	7.0e-06	Thermal expansion coefficient
E	GPa	22	Young's modulus of elasticity
P3	kN/m^2	2900	Uniaxial tensile strength
m		1	Parameter for curvature of the yield surface towards the hydrostatic axis
η		88162	Yield function
f_{cd}	kN/m^2	33333	Compressive strength
ε_{tu}	‰	0.2	Tensile failure strength

3.2 *Discretization of dam body and foundation by finite elements*

Numerical analysis of the arch is carried out by spatial (3D) model, where the dam body and the foundation are modeled with volume elements, by full reproduction of the finite element model formulation data. A powerful and reliable finite element should be applied in case where an analysis of structure with complex geometry and behavior is required, having in consideration that the correctly calculated deformations and stresses are of primary significance for assessment of the dam stability. In this case, for discretization of the dam body and the rock foundation are applied finite element type bric, by 4 nodes, identical to C3D4 element from ABAQUS and kinematic constraints at the interface dam-rock foundation. Namely, the model is composed of dam body and rock foundation with constraints at the interface dam-foundation.

The spatial (3D) model has geometrical boundaries, limited to horizontal and vertical plane. In these planes are defined the boundary condition of the model (Figure 2). The curvature plane in the lowest zone of the model is adopted as non-deformable boundary condition (fixed displacements in XYZ direction), vertical planes perpendicular on X-axis are boundary condition by applying fixed (zero) displacements in X-direction and vertical planes perpendicular on Y-axis are boundary condition by applying fixed (zero) displacements in Y-direction. The discretization is conveyed by including zones of various materials in the model – concrete and rock foundation. The dam is modeled as monolithic structure.

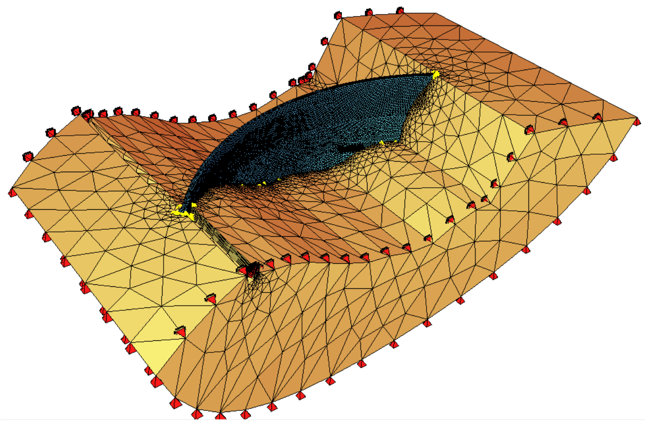

Figure 2: View of the numerical model, discretized with total of 186583 elements and 39419 nodes.

3.3 *Dam loading scenarios for calibration and prognosis stage*

The dam loading is directly correlated with the calibration and prognosis stage for the dam behavior. The numerical analysis is carried out by coupled thermo-mechanical model and hydraulic (seepage) model for analysis of the dam behavior in the calibration and prediction stage. The thermal effect of the dam is simulated by applying temperature loading of the dam body according to air temperature time series as uniform distribution within the volume (bric) elements. The assumption is that the temperature loading in the dam body is uniformly distributed in the various time steps and approximately in range of the air temperature. The temperature effect is coupled with the hydrostatic loading in accordance with the specified water levels from the reservoir for the identical time steps. The hydrostatic loading is applied at upstream face of the dam as spatial load in accordance with the water levels in the reservoir. The calibration process is carried out by combined choice of extreme (highest) values for the air temperature Ta, water levels in the reservoir WL and measured values for the variables records within the monitoring process. On Fig. 3 is displayed the method of adoption for only five extreme values per dates for time series of pendulums CB2. In such a way afterwards by analogy is are adopted records for the water levels and temperature, thus obtaining a representative number of load cases in order to assess the dam behavior, that are being run within the model.

Identical approach is applied and for the prognosis stage. Namely, for the short-term prognosis is carried out calculation of the dam response for all time steps from January-June, 2013 (total of 184), while for the long-term prognosis is applied also combined choice of extreme values for period July, 2013-December, 2017, regarding the water levels in the reservoir and temperature Ta, thus obtaining total a representative number of load cases for the various variables with aim to predict the dam's behavior, that are being run within the model. Static loading scenarios includes self-weight of the concrete arch dam and the rock foundation.

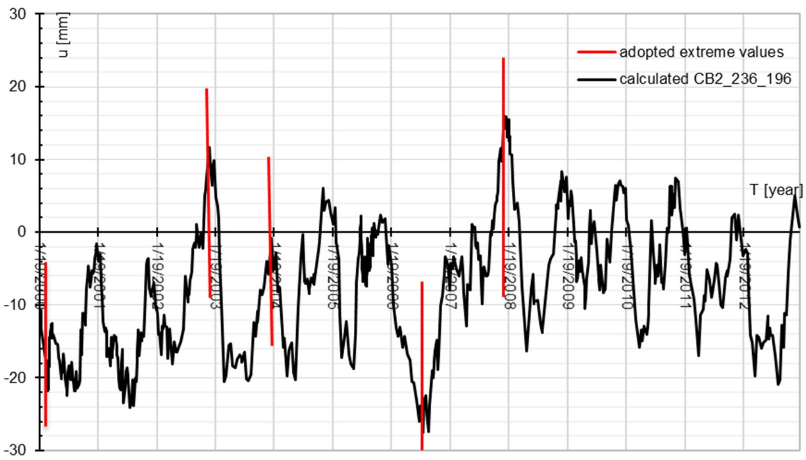

Figure 3: Display of approach for combined adoption of extreme values for calibration process.

4 CALIBRATION PROCESS OF THE DAM

The calibration process includes analysis of variables such as radial displacements (pendulums CB2 and CB3 in the central block of the dam), crack opening displacement (sensor C4-C5 at rock foundation interface), piezometric levels (PZCB2 and PZCB3 at rock-foundation interface) and seepage (weir at the downstream toe of the dam). The required calculated data are derived for corresponding nodes within the numerical model.

4.1 *Calibration of pendulums displacements time series*

The calibration process is carried out by comparison of the measured and calculated radial displacements at corresponding nodes of the model respectively. On Fig. 4 are displayed calculated radial of the dam for maximal registered water level H=234.95 m. The radial displacements are mainly in downstream direction, with some section in upstream direction towards the right bank.

By comparison of the radial displacements for pendulum CB2 (Fig. 5) it can be noticed good matching of the data regarding the distribution and the values. In case of radial displacements for pendulum CB3 (Fig. 6) it can be noticed good matching of the data regarding the distribution and some less good matching od the data regarding the values. Namely, the calculated radial displacements are mainly lower than the measured values although they have very similar distribution and shape within the calibration timeline. The lower calculated values for the radial displacements can result from the behavior of the interface zone dam-foundation, that is modeled by applying kinematic constraints that enable stiff coupling connection. The maximal measured values for pendulum CB2 range in interval from 15.95 mm to -27.48 mm, while the calculated values from 19.5 mm to -22.9 mm. In case of pendulum CB3 the maximal measured values for pendulum CB2 range in interval from 3.91 mm to -5.0 mm, while the calculated are from 2.5 mm to -0.47 mm.

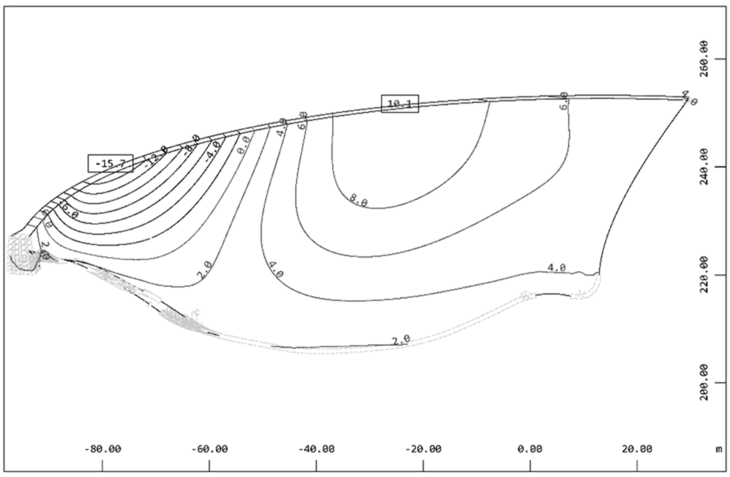

Figure 4: Calculated radial displacements of the dam for water level H=234.95 m.

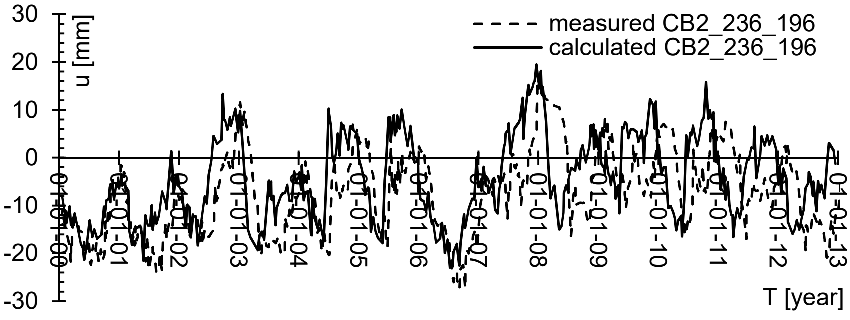

Figure 5: Display of measured and calculated time series of CB2 pendulums displacements for 2000-2012.

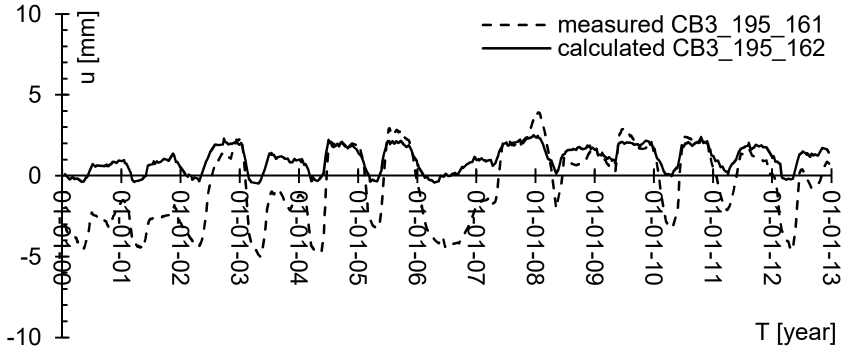

Figure 6: Display of measured and calculated time series of CB3 pendulums displacements for 2000-2012.

The measured and calculated pendulums displacements time series are generally in correlation with the variation of the water level in the reservoir and air temperature. Namely, at higher water levels in the reservoir the displacements are in downstream direction (the hydrostatic pressure generates greater displacement then the temperature effect), while at lower water levels in the reservoir the displacements are in upstream direction, combined with the temperature effect that generates displacements in upstream direction.

4.2 *Calibration of crack opening time series*

The calibration process for the crack opening displacements is carried out by numerical model with nonlinear constitutive law for concrete (Table 2). The calculated radial displacements as deduction of the radial displacements in the corresponding nodes are projected to sensor C4-C5 direction in order to obtain the variation values for the relative distance. On Fig. 7 are displayed radial displacements in the dam for water level H=234.95 m. The displacements are mainly in downstream direction, with maximal value of 11 mm at the crest. On Fig. 8 are displayed calculated and measured relative distance for displacements in direction of the sensor C4-C5. It can be noticed that in general there is a similar distribution of the values for the calibration period, however there is a less good matching of the calculated and measured values. Namely, the calculated relative distance values are mainly lower them the measured values. The reason for such lower degree of matching of the values can be the stiff coupling condition at interface dam-foundation, modeled as kinematic constraint in the model. So a potential case to be investigated is to model the contact dam-foundation by interface elements (with both linear and non-linear properties) combined with the variation of the stiffness properties of the rock (in central part and the banks) in order to improve the calibration process.

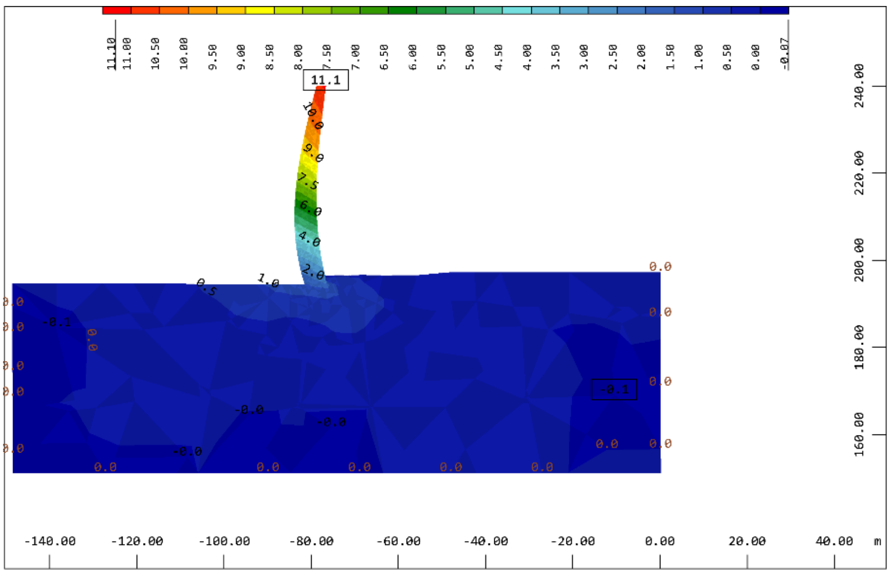

Figure 7: Display of radial displacements at central block for water level H=234.95 m.

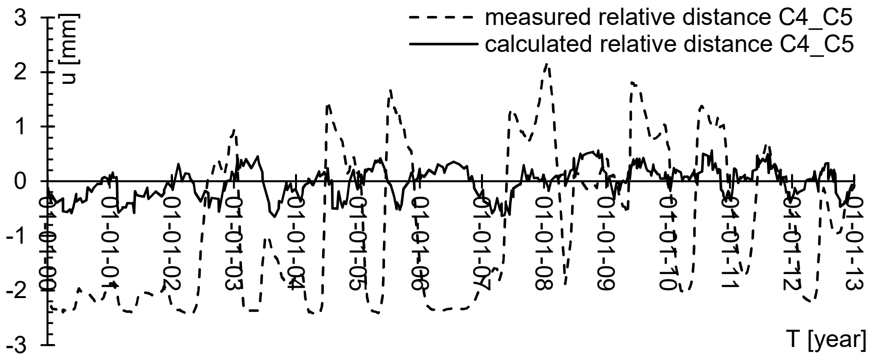

Figure 8: Display of calculated and measured time series of crack openings at sensor C4-C5 for 2000-2012.

The measured and calculated values are mainly in reverse correlation with the water level in the reservoir apropos in period when the water level is low there is increase of the relative distance (positive values) while in period of higher water levels there is a decrease (negative values). Regarding the temperature, the displacement manifest more variable behavior apropos the applied temperature effect has lower influence the water level in the reservoir. The maximal measured relative distance values range in interval (2.17 ÷ -2.43) mm, while the calculated values vary in range (0.56 ÷ -0.65) mm.

4.3 *Calibration of piezometric levels and seepage time series*

The calibration process is carried out by plane (2D) numerical model for seepage analysis by modelling the foundation medium below the central block of the dam including running of load cases for full timeline period 2000-2012, and subsequent comparison of the measured and calculated piezometric levels at corresponding nodes for piezometers PZBC2 and PZBC3. The calculated piezometric levels are obtained by the values of the equipotential lines (H).

The first step is to calibrate the value for the permeability coefficient k in accordance with the seepage values. The assessed permeability coefficient foe laminated metamorphic slate ranges in interval $k=(10^{-6}\div 10^{-9})$m/s (Lianyang, 2016). The permeability coefficient can be calibrated by the value of the full seepage flow directly below the dam, specified as measured values in weir at gallery located at the downstream toe of the dam. So, according to the measuring data for water level at 232.0 m the registered seepage at the weir is 8.8 l/min. Accordingly, seepage analysis was carried out for H=232.0 m as upstream boundary condition and H=0 m as downstream boundary condition, by applying Darcy flow rule adopting the rock foundation as homogeneous flow medium and assumed permeability coefficient in first iteration $k=1\times10^{-7}$ m/s. By the calibration of the permeability coefficient was obtained value of $k=8.3\times10^{-8}$ m/s, applied in the calculation for the full calibration and prognosis analysis of the piezometric levels and seepage. For the calibration and prognosis analysis is adopted flow medium below the dam including vertical grout curtain below the dam, by assuming higher permeability coefficient with value of $k_{gc}=8.3\times10^{-9}$ m/s. The obtained equipotential lines flow net and contour lines of flow quantities in the rock foundation for water level H=211.4 m are displayed on Fig. 9.

By comparison of the piezometric levels for piezometers PZCB2 (Fig. 10) it can be noticed good matching of the measured and calculated data regarding the distribution however there is a difference in the obtained values (the readings from the equipotential lines for calculated values are higher than the measured values) pointing out that there is necessity for additional improvement of the numerical model and calibrated value of the permeability coefficient. Namely, the seepage medium is required to be modeled by taking in consideration the full geometry (vertical and inclined zone) and the hydraulic properties of the grout curtain. However, in general the applied assumptions and the seepage calculation model are chosen correctly. The piezometric levels are in accordance with the variation of the water level in the reservoir. The maximal measured piezometric levels varies in interval (195.0 ÷ 210.5) mm, while the calculated values vary in interval (197.5 ÷ 217.0) mm.

By comparison of the calculated and measure seepage values (Fig. 11) it can be noticed good matching of the data regarding the distribution, but in case of the seepage flow values there is a less matching, apropos the calculated values are mainly lower than the measured values. Such case indicates to additional calibration of the permeability coefficient. The measured peak values of the seepage flow occur approximately at normal water level so this may be indication for additional leakage occurrences that affect the seepage process. The maximal measured seepage flow varies in interval (0.01 ÷ 26.5) l/min, while the calculated values in interval (0.001 ÷ 11.7) l/min.

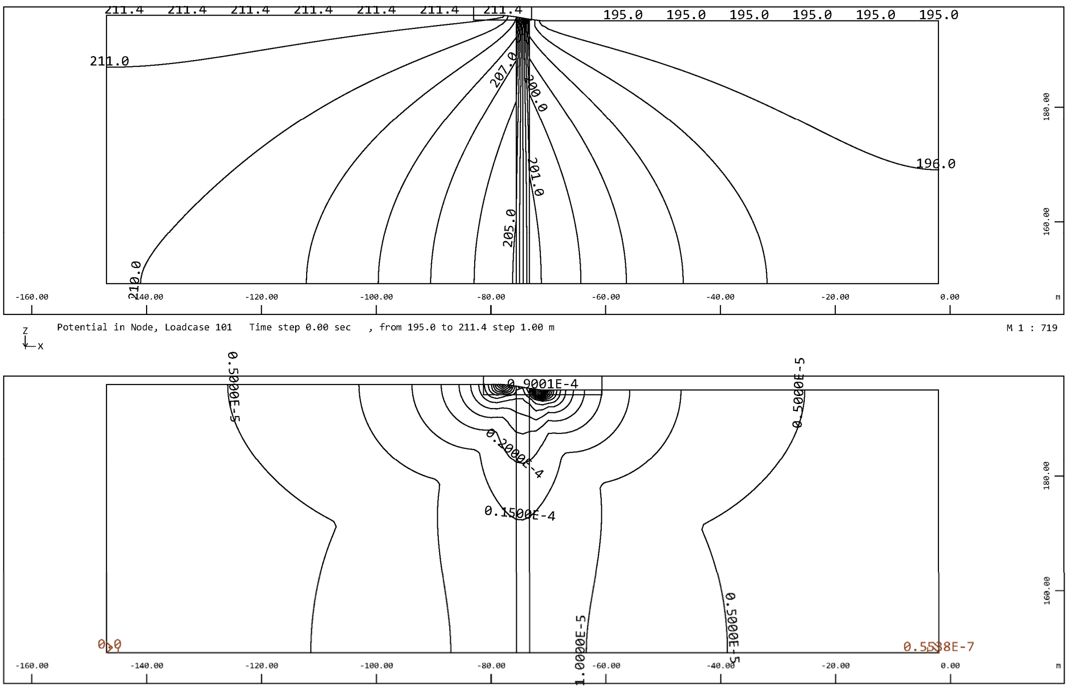

Figure 9: Equipotential lines (upper) and contour lines of flow quantity (lower) in the foundation medium for water level H=211.4 m.

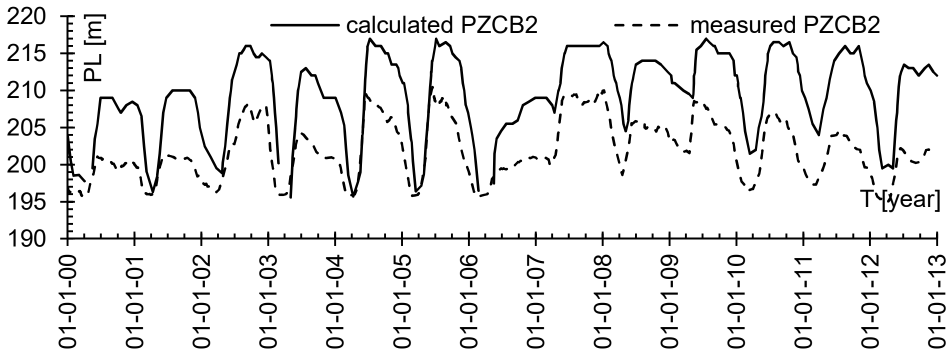

Figure 10: Display of measured and calculated piezometric levels for PZCB2 for period 2000-2012.

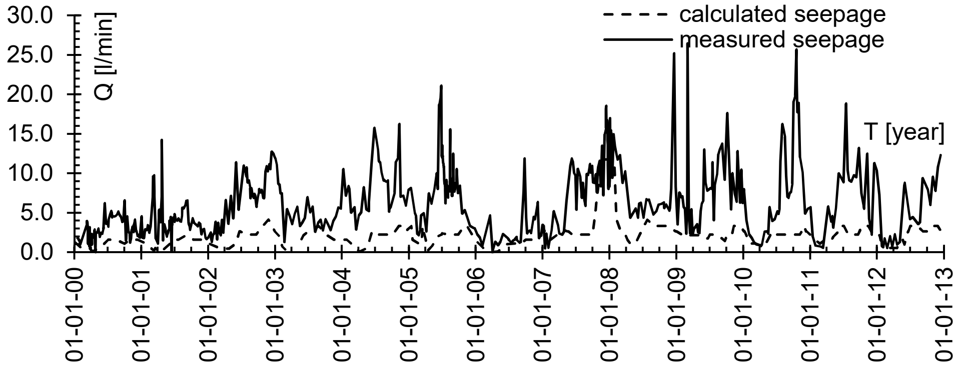

Figure 11: Display of measured and calculated seepage flow time series at the weir for period 2000-2012.

5 PREDICITON (PROGNOSIS) PROCESS OF THE DAM

5.1 *Prognosis of pendulums displacements time series*

The prognosis stage consists of short-term and long-term prediction of the specified variables. Namely, the short-term prediction includes period January, 2013-June, 2013, while the long-term prediction captures period July, 2013-December, 2017. The prognosis process is carried out by numerical model including running of full time steps for the time series within the short-term period. The prognosis process for the long-term behavior is carried out by numerical model including running of appropriate representative load cases for the time series.

The calculated displacements for pendulum CB2 (Fig. 12) for the short-term prognosis are mainly in upstream direction, maximal value of -16.3 mm, while the maximal displacement in the downstream direction is 1.75 mm. The calculated displacements for pendulum CB3 for short-term period are mainly in downstream direction, with maximal value of 1.7 mm and maximal value of -0.3 mm in downstream direction. The calculated values for pendulum CB2 are in correlation with the water level apropos the lowering of the water level within the short-term period is dominant factor that enables modus for manifestation of the displacements in the upstream direction while the applied temperature effect has lower influence on the displacements. In case of pendulum CB3 the lowered water level and temperature does not generate significant variation of the displacements values.

The calculated displacements for pendulum CB2 (Fig. 12) for the long-term prognosis are also mainly in upstream direction, maximal value of -24.6 mm, while the maximal displacement in the downstream direction is 10.5 mm. The calculated displacements for pendulum CB3 for the long-term period are mainly in downstream direction, with maximal value of 2 mm and maximal value of -0.55 mm in downstream direction. The calculated values for pendulum CB2 are mainly in correlation with the water level apropos the lowered water level within the long-term period is contributing factor that enables modus for manifestation of the displacements in the upstream direction while the applied temperature effect has lower influence on the displacements. In case of pendulum CB3 the lowered water level and temperature does not generate significant variation of the displacements values.

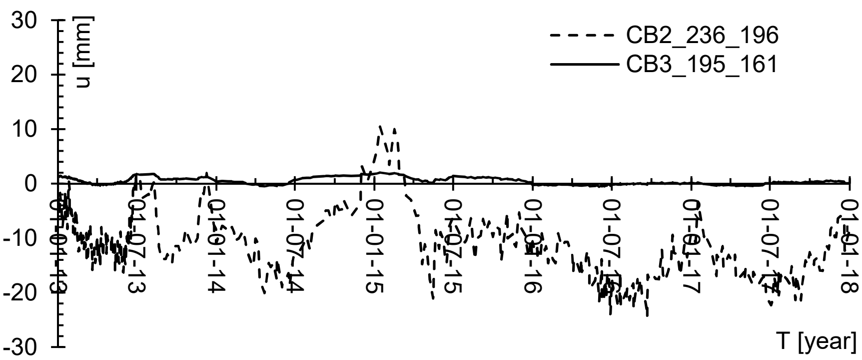

Figure 12: Display of calculated prognosis time series of CB3 pendulums displacements for January, 2013-December, 2017.

5.2 *Prognosis of crack opening time series*

The prognosis process is carried out by numerical model including running of full time steps for the time series within the short-term period. The prognosis process for the long-term behavior is carried out by numerical model including running of appropriate representative load cases for the time series. The calculated relative distance for the short-term prognosis at sensor C4-C5 are varying from increase (positive values) to decrease (negative) values (Fig. 13). The maximal calculated values are -0.5 mm and 0.3 mm respectively. The calculated values are in correlation with the water level apropos at higher water level the crack displacements are increasing while at lowered water level they are decreasing. Very similar case is also with the applied temperature effect apropos lowering of the temperature in the short-term period lowers the crack displacements. The calculated relative distance for the long-term prognosis at sensor C4-C5 are less varying from increase (positive values) and decrease (negative) values apropos mainly manifest decrease of the crack openings. The maximal calculated values are -0.72 mm and 0.56 mm respectively. The calculated values are in correlation with the water level apropos at longer period of higher water level the crack displacements are increasing while at lowered water level they are decreasing. Very similar case is also with the applied temperature effect apropos lowering of the temperature in the long-term period lowers the crack displacements.

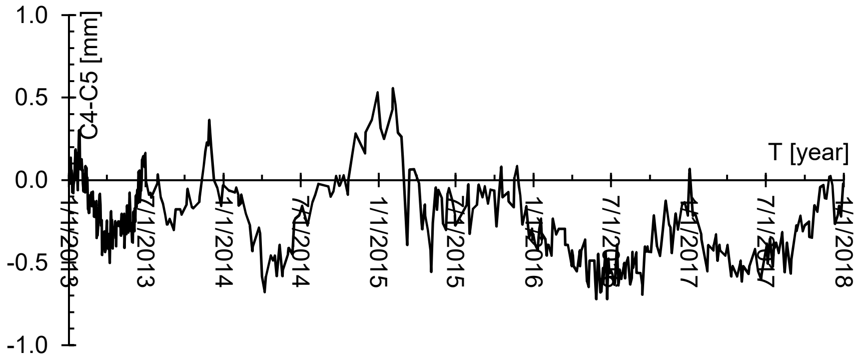

Figure 13: Prognosis calculated time series of C4-C5 relative distance for January, 2013-December, 2017.

5.3 *Prognosis of piezometric levels time series*

The prognosis process for piezometric levels for PZCB2 is carried out by numerical model for seepage analysis including running of full time steps for the time series within the short-term and the long-term period (Fig. 14). The calculated piezometric levels for piezometer PZCB2 for the short-term and long term prognosis, as expected, are varying in correlation with the water level in the reservoir (Fig. 15). The maximal and minimal calculated values for the piezometric levels are respectively 213.0 m and 195.5 m.

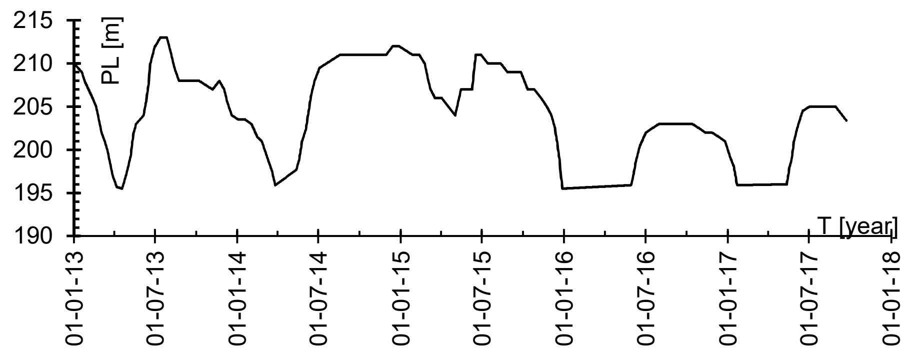

Figure 14: Prognosis calculated time series of piezometric levels for PZCB2 for Jan,2013-Dec,2017.

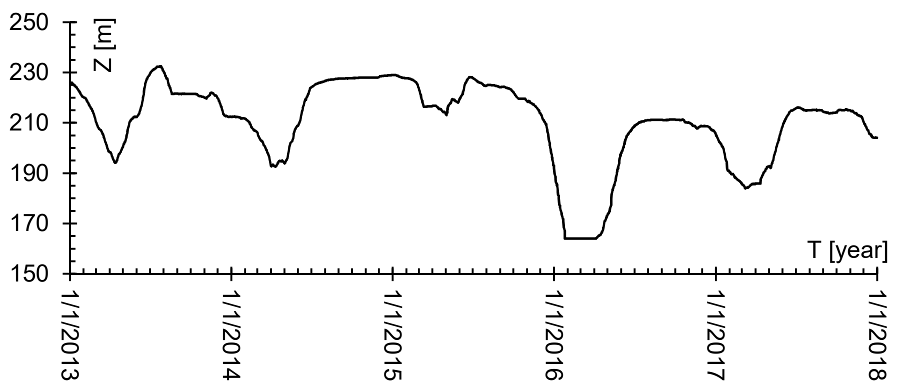

Figure 15: Variation of water levels in the reservoir for Jan,2013-Dec,2017.

5.4 *Prognosis of seepage time series*

The calculated seepage flows for the short-term and long term prognosis, as expected, are varying in correlation with the water level in the reservoir (Fig. 16). The maximal and minimal calculated values for the seepage flow are respectively 3.34 l/min m and 0.06 l/min.

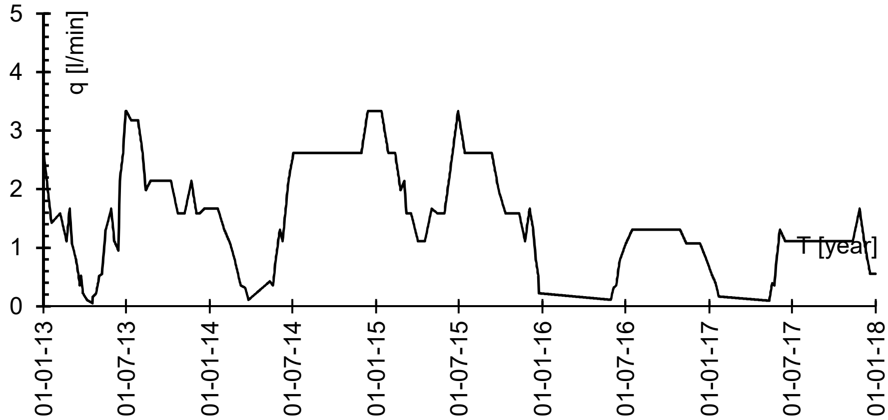

Figure 16: Prognosis calculated time series of seepage flow for Jan,2013-Dec,2017.

6 CONCLUSIONS

The behavior of the dam during the service period for variation of the water levels in the reservoir and temperature effect was simulated by application of the Finite Element Method with spatial (3D) numerical model. The numerical analysis was carried out by taking in consideration the specified data for the numerical model (fully reproduced according to the formulation data) and variations of the water level in the reservoir and the air temperature, by applying coupled thermo-mechanical analysis of the dam in static conditions. The loading scenarios for calculation of the required variables were adopted according to the extreme values for the available monitoring records of the variables and records for reservoir water levels and air temperature by coupled thermo-mechanical analysis of the dam and hydraulic analysis of the seepage process.

The prediction of the behavior of the dam was analyzed in two stages – calibration and prognosis stage. From the carried out numerical experiment of simulation for prediction of the behavior of the dam EDF, following main conclusions and recommendations are derived:

The calibration process of the measured and calculated radial displacements for pendulum CB2 provided good matching of the data regarding the distribution and the values. In case of pendulum CB3 a good matching of the data regarding the distribution and some less good matching od the data regarding the values is obtained.

The calibration process for the relative distance C4-C5, in general, has a good matching of the distribution of the values, however there is a difference in the calculated and measured values (the calculated relative distance values are mainly lower them the measured). The measured and calculated values are mainly in reverse correlation with the water level in the reservoir apropos in period when the water level is low there is increase of the crack opening (positive values) while in period of higher water levels there is a decrease (negative values). Regarding the temperature, the displacements manifest less variable behavior apropos the applied temperature has lower influence on the displacements.

By comparison of the calculated and measured piezometric levels for piezometers PZCB2 and PZCB3 as well and the seepage flow is obtained good matching of the records regarding the distribution and less good matching regarding the values.

The calculated displacements for pendulum CB2 prognosis period are mainly in upstream direction, while the calculated displacements for pendulum CB3 are mainly in downstream direction. The calculated values for pendulum CB2 are mainly in correlation with the water level apropos the lowered water level while the applied temperature effect has lower influence on the displacements. In case of pendulum CB3 the lowered water level and temperature does not generate significant variation of the displacements values.

The calculated relative distance for the prognosis period at sensor C4-C5 are varying from increase (positive values) to mainly decrease (negative) values. The calculated values are in correlation with the water level apropos at higher water level the crack displacements are increasing while at lowered water level they are decreasing. Very similar case is also with the applied temperature effect apropos lowering of the temperature in the short-term period lowers the relative distance C4-C5.

The calculated piezometric levels for piezometer PZCB2 and PZCB3 and seepage flow for the short-term and long term prognosis, are varying in correlation with the water level in the reservoir.

Improved calibration should be carried out apropos the case to be analyzed is to model the contact dam-foundation by interface elements (with both linear and non-linear properties) combined with the variation of the stiffness properties of the rock (in central part and the banks).

In case of the seepage analysis, next calibration iteration is to model in full the grout curtain and to update the permeability coefficient value.

The overall behavior of the concrete arch dam, taking in consideration the findings from the calibration and the prognosis stage, is within the expected mode for such structure. Namely, according to the carried out thermo-mechanical and seepage analysis of the system dam-foundation it can be concluded that the variation of the obtained values for displacements and seepage flow are in general in good correlation with the variation of the water level in the reservoir and the air temperature.

According to the measured and calculated values for the variables at this stage an interval variation can be specified (greater calculated/measured value) for proper mode of behavior of the dam. A more systematic approach to specify the warning levels for the dam behavior should be conveyed in correlation with stress state of the dam for static and dynamic loading.

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